Exercise 67

Find a second-degree polynomial P such that $P(2) = 5$, $P'(2) = 3$, and $P''(2) = 2$.

Solution

The general form of a second-degree polynomial is

$$P(x) = ax^2 + bx + c.$$

Its first derivative is

$$P'(x) = 2ax + b,$$

and its second derivative is

$$P''(x) = 2a.$$

Use the given formulas to obtain a system of equations for the unknowns, a and b and c.

$$\begin{aligned} P(2) &= a(2)^2 + b(2) + c = 5 \\ P'(2) &= 2a(2) + b = 3 \\ P''(2) &= 2a = 2 \end{aligned}$$

Simplify the system

$$\begin{aligned} 4a + 2b + c &= 5 \\ 4a + b &= 3 \\ 2a &= 2 \end{aligned}$$

and then solve it.

$$a = 1 \qquad b = -1 \qquad c = 3$$

Therefore, the second-degree polynomial is

$$P(x) = x^2 - x + 3.$$